Simultaneous Hypothesis Testing for Parameters of Spline Truncated Nonparametric Regression for Longitudinal Data

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Abstract— In multivariable regression, response variables can have a particular pattern of relationships with predictor variables one of which has an unknown pattern of curve form. The nonparametric regression approach is an appropriate approach for the case. One of the nonparametric regression approaches is the spline truncated, which has the advantage of knot points. With the point of knots, the resulting model will follow the form of changes in data behavior patterns. Changes in data behavior patterns if observed over and over can provide more complete information about the dynamics of changes in the behavioral patterns of the data. Data obtained from the repeated observation of each object at different time intervals is called longitudinal data. Longitudinal data is data from observations and measurement of the same individual at certain time periods, which is different from cross section data where the data from each individual is observed only once. In a statistical analysis, in addition to using descriptive statistics, it is also necessary to conduct an inferential statistical analysis because it is very important to do, one of them is testing the simultaneous hypothesis of the model parameters to determine whether the parameter is significant to the model. In this paper conducted a study of simultaneous hypothesis testing of parameters on spline truncated nonparametric regression model especially on longitudinal data. Based on the result of the simultaneous hypothesis testing of parameters on spline truncated nonparametric regression model, it is found that the distribution of test model statistic follows the distribution F with degrees of freedom ((np+nrp), ntp-(np+nrp)).

Index Terms— nonparametric regression, spline truncated, longitudinal data, simultaneous hypothesis testing.

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1 Introduction

Regression analysis is a statistical method who determine relationship pattern between predictor variables and response variable. The aim of this regression is to estimed parameters who matched with the regression curved. The estimation of regression curve form is used to explain the relationship between response variables and predictor variables. One of the most commonly used approaches is the parametric regression approach, where the assumption underlying this approach is that the form of the regression curve can be represented by a set of certain parameters or can be described in a particular pattern [1]. When the model of regression curve is unknown, then nonparametric regression analysis is preferred to used [2]. One of the nonparametric regression approaches is the spline truncated, which has the advantage of knot points. With the point of knots, the resulting model will follow the form of changes in data behavior patterns [3].

Changes in data behavior patterns if observed over and over can provide more complete information about the dynamics of changes in the behavioral patterns of the data. Data obtained from the repeated observation of each object at different time intervals is called longitudinal data. Research by nonparametric approach on longitudinal data is mostly done in the field of health, but also can be applied to the others, including the social and economic [4]. Longitudinal data is data from observations and measurement of the same individual at certain time periods, which is different from cross section data where the data from each individual is observed only once [5]. In a statistical analysis, in addition to using descriptive statistics, it is also necessary to conduct an inferential statistical analysis because it is very important to do, one of them is testing the simultaneous hypothesis of the model parameters to determine whether the parameter is significant to the model.

Simultaneous hypotesis testing is one of the most important parts of statistical inference. Simultaneous hypotesis testing for parameters of nonparametric regression can be used to find out if predictor variables that significantly influence response variables. From a statistical test will be obtained a region of rejection that will be used to generate a decision for the results of simultaneous hypothesis testing. If p-value is less then α, then at least one of the predictor variable has significant effect on the response variable. In this research will study about simultaneous hypothesis testing in *spline truncated* nonparametric regression for longitudinal data.

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2 Literature review

2.1 Nonparametric Regression Model

The regression curve between predictor and response variables is not always known. If forced to use parametric regression then the resulting model is not in accordance with the form of relationship pattern which will ultimately produce a large error.

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Nonparametric regression is one of the approaches used to determine the relationship pattern between predictor variables and the unknown response of the regression curve or no complete past information about the shape of the data pattern [3].

Some approaches in nonparametric regression include: spline, kernel, fourier series, wavelet, etc. Spline is an approach often used in nonparametric regression. Somethings to consider informing spline regression is to determine the order of the model, the number of knots, and the location of the knot point [6]. Spline regression has a functional basis which in its parameter optimization process uses optimization. Spline regression has the advantage of adjusting data patterns that change sharply with knots. In general, nonparametric regression model:

$$y_i = f(z_i) + \varepsilon_i, \ \ i = 1,2,\dots,n\,, \tag{1}$$

with y_i is the i -th response variable, while the function $f(z_i)$ is the regression curve, with z_i as the predictor variable and ε_i is the random error assumed to be independent normal distribution with mean zero and variance σ^2 [3].

2.2 Spline Truncated Nonparametric Regression for Longitudinal Data

Spline Truncated nonparametric regression model on longitudinal data can be written in the form:

$$y_{ij} = f(z_{ij}) + \varepsilon_{ij}\,, i = 1,2,\dots,n; j = 1,2,\dots,t \tag{2}$$

with

$$f(z_{ij}) = \sum_{q=1}^{m} \beta_{qi} z_{ij}^{q} + \sum_{l=1}^{r} \gamma_{li}(z_{ij} - K_{li})_{+}^{m},$$

where n is the number of observed objects and t is the amount of time of the object being observed, while $\sum_{q=1}^{m} \beta_{qi} z_{ij}^{q}$ is polynomial components dan $\sum_{l=1}^{r} \gamma_{li}(z_{ij} - K_{li})_{+}^{m}$ is truncated *truncated* with:

$$(z_{ij} - k_{li})_{+}^{m} = \begin{cases} (z_{ij} - k_{li})^{m}, & z_{ij} \geq k_{li} \\ 0, & z_{ij} < k_{li} \end{cases}$$

Equation (1) is a nonparametric regression spline-truncated form in longitudinal data with one nonparametric predictor variable. If the nonparametric spline truncated regression in the longitudinal data consists of one response variable with a nonparametric predictor variable of q, then the spline truncated regression curve for longitudinal data with m = 1 can be expressed in terms of the following equation

$$f(z_{ij}) = \sum_{k=1}^{p}\left(\sum_{q=1}^{m} \beta_{qi} z_{ijk}^{q} + \sum_{l=1}^{r} \gamma_{li}(z_{ijk} - k_{li})_{+}^{m}\right).$$

So, equation (2) becomes

$$y_{ij} = \sum_{k=1}^{p}\left(\sum_{q=1}^{m} \beta_{qi} z_{ijk}^{q} + \sum_{l=1}^{r} \gamma_{li}(z_{ijk} - k_{li})_{+}^{m}\right) + \varepsilon_{ij}\,, \tag{3}$$

with $i = 1,2,\dots,n; j = 1,2,\dots,t.$

In longitudinal data, parameter estimates were obtained using *Weighted Least Square* (WLS) to overcome correlations in the same observational subjects. Then write equation (3) in matrix notation as follows

$$\tilde{y} = T(Z, \boldsymbol{Z}[\mathbf{k}])\tilde{\delta} + \tilde{\varepsilon} \tag{4}$$

the $\tilde{\delta}$ consist of the parameter $\hat{\tilde{\beta}}$ dan $\hat{\tilde{\gamma}}$. Estimator of $\hat{\tilde{\delta}}$ is obtained by completing the WLS optimization as follows:

$$\min_{\tilde{\delta} \in R^{(m+r)np}} \{(\tilde{y} - T(Z, \boldsymbol{Z}[\mathbf{k}])\tilde{\delta})'\mathbf{W}(\tilde{y} - T(Z, \boldsymbol{Z}[\mathbf{k}])\tilde{\delta})\}, \tag{5}$$

With matrix $\boldsymbol{W}$ is given by

$$\boldsymbol{W} = \begin{bmatrix} \boldsymbol{W_1} & 0 & \cdots & 0 \\ 0 & \boldsymbol{W_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \boldsymbol{W_n} \end{bmatrix},$$

2.3 Generalized Cross Validation (GCV)

One of the most commonly used methods of choosing an optimum knot point is the Generalized Cross Validation (GCV). Compared with other methods, such as Cross Validation (CV) and Unbiased Risk (UBR) or Generalized Maximum Likelihood (GML) methods, GCV has theoretically optimal asymptotic properties [7]. GCV method also has advantages that do not require knowledge of the population variance σ^2 and GCV invariance method of transformation [7].

GCV function is given by

$$GCV(\tilde{k}) = \frac{n^{-1}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\left[1 - n^{-1} trace(A(\tilde{k}))\right]^2},$$

with GCV being a vector containing GCV values from knot points. Optimum knot points is obtained through optimization

$$\min_{k_1,k_2,\dots,k_r}\{GCV(\tilde{k})\} = \min_{k_1,k_2,\dots,k_r}\left\{\frac{n^{-1}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\left[1 - n^{-1} trace(A(\tilde{k}))\right]^2}\right\},$$

with $\tilde{k} = (k_1, k_2, \dots, k_r)$. $\boldsymbol{A}[\tilde{k}]$ get from equation

$$\hat{\tilde{y}} = \boldsymbol{A}[\tilde{k}]\tilde{y}$$

3 Methodology Research

3.1 Source of Data

This research used secondary data from Gross Regional Domestic Product (GRDP) by industry classification and Regency/Municipality in Papua Province in 2011-2016, and Human Development Index (HDI) in 2011-2016. The observation units are 29 Regency/Municipality in Papua Province.

3.2 Variables Research

The research variables used is response variables (Y) and two predictor variables (Z_1 *dan* Z_2).

TABLE 1.
RESEARCH VARIABLES

Variabels	Explanation
$\boldsymbol{Y}$	Economic Growth Rate
$\boldsymbol{Z_1}$	Mean Years of Schooling (MYS)
$\boldsymbol{Z_1}$	Life Expectancy – e_0

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4 Results and Discussion

4.1 Estimation of Spline Truncated Regression Nonparametric Parameters on Longitudinal Data

Based on equation (4) and (5) we get estimator of parameter $\hat{\tilde{\beta}}$

$$\hat{\tilde{\beta}} = \left\{\mathbf{I} - (\boldsymbol{Z}^T\boldsymbol{W}\mathbf{Z})^{-1}\boldsymbol{Z}^T\boldsymbol{W}\mathbf{Z}(\tilde{k})\left(\boldsymbol{Z}(\tilde{k})^T\boldsymbol{W}\mathbf{Z}(\tilde{k})\right)^{-1}\boldsymbol{Z}(\tilde{k})^T\boldsymbol{W}\mathbf{Z}\right\}^{-}$$
$$(\boldsymbol{Z}^T\boldsymbol{W}\mathbf{Z})^{-1}\boldsymbol{Z}^T\boldsymbol{W}\tilde{y} - (\boldsymbol{Z}^T\boldsymbol{W}\mathbf{Z})^{-1}\boldsymbol{Z}^T\boldsymbol{W}\mathbf{Z}(\tilde{k})\left(\boldsymbol{Z}(\tilde{k})^T\boldsymbol{W}\mathbf{Z}(\tilde{k})\right)$$
$$\boldsymbol{Z}(\tilde{k})^T\boldsymbol{W}\tilde{y}$$

$$\hat{\tilde{\beta}} = \mathbf{U}(\tilde{k})\tilde{y}$$

with $\mathbf{U}(\tilde{k}) = \mathbf{A}(\boldsymbol{Z}^T\boldsymbol{W}\mathbf{Z})^{-1}\left\{\boldsymbol{Z}^T\boldsymbol{W} - \boldsymbol{Z}^T\boldsymbol{W}\mathbf{Z}(\tilde{k})\left(\boldsymbol{Z}(\tilde{k})^T\boldsymbol{W}\mathbf{Z}(\tilde{k})\right)^{-1}\boldsymbol{Z}(\tilde{k})^T\boldsymbol{W}\right\}$

and we get

$$\hat{\tilde{\gamma}} = \left\{\mathbf{I} - \left(\boldsymbol{Z}(\tilde{k})^T\boldsymbol{W}\mathbf{Z}(\tilde{k})\right)^{-1}\left(\boldsymbol{Z}(\tilde{k})^T\boldsymbol{W}\mathbf{Z}\right)\boldsymbol{Z}^T\boldsymbol{W}\mathbf{Z}(\tilde{k})\right\}^{-1}$$
$$\left(\boldsymbol{Z}(\tilde{k})^T\boldsymbol{W}\mathbf{Z}(\tilde{k})\right)^{-1}\boldsymbol{Z}(\tilde{k})^T\boldsymbol{W}\tilde{y} - \left(\boldsymbol{Z}(\tilde{k})^T\boldsymbol{W}\mathbf{Z}(\tilde{k})\right)^{-1}$$
$$\boldsymbol{Z}(\tilde{k})^T\boldsymbol{W}\mathbf{Z}(\boldsymbol{Z}^T\boldsymbol{W}\mathbf{Z})^{-1}\boldsymbol{Z}^T\boldsymbol{W}\tilde{y}$$

$$\hat{\tilde{\gamma}} = \mathbf{V}(\tilde{k})\tilde{y}$$

with $\mathbf{V}(\tilde{k}) = \mathbf{A}\left(\boldsymbol{Z}(\tilde{k})^T\boldsymbol{W}\mathbf{Z}(\tilde{k})\right)^{-1}\left\{\boldsymbol{Z}(\tilde{k})^T\boldsymbol{W} - \boldsymbol{Z}(\tilde{k})^T\boldsymbol{W}\mathbf{Z}(\boldsymbol{Z}^T\boldsymbol{W}\mathbf{Z})^{-1}\boldsymbol{Z}^T\boldsymbol{W}\right\}$

After get estimator $\hat{\tilde{\beta}}$ dan $\hat{\tilde{\gamma}}$, then the spline truncated regression model on the longitudinal data can be written to be:

$$\begin{aligned}\hat{\tilde{y}} &= \mathbf{Z}\hat{\tilde{\beta}} + \mathbf{Z}(\tilde{k})\hat{\tilde{\gamma}} \\ &= \mathbf{ZU}(\tilde{k})\tilde{y} + \mathbf{Z}(\tilde{k})\mathbf{V}(\tilde{k})\tilde{y} \\ &= \left(\mathbf{ZU}(\tilde{k}) + \mathbf{Z}(\tilde{k})\mathbf{V}(\tilde{k})\right)\tilde{y} \\ &= \left(\mathbf{M}(\tilde{k}) + \mathbf{N}(\tilde{k})\right)\tilde{y} \\ &= \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}\end{aligned}$$

with:

$$\mathbf{M}(\tilde{k}) = \mathbf{ZU}(\tilde{k})$$
$$\mathbf{N}(\tilde{k}) = \mathbf{Z}(\tilde{k})\mathbf{V}(\tilde{k})$$

4.2 Hypothesis Testing Formulation

Give a spline truncated nonparametric regression model on longitudinal data in matrix form as follows:

$$\tilde{y} = \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta} + \tilde{\varepsilon} \qquad (6)$$

with

$$\tilde{y} = \begin{bmatrix}\tilde{y}_1\\ \tilde{y}_2\\ \vdots\\ \tilde{y}_n\end{bmatrix}_{nt\text{ x }1};\mathbf{T} = [\mathbf{Z}\quad \mathbf{Z}(\tilde{k})]_{nt\text{ x }(np+nrp)};\tilde{\delta} = \begin{bmatrix}\tilde{\beta}\\ \tilde{\gamma}\end{bmatrix}_{(np+nrp)\text{ x }1}$$

$$\tilde{\varepsilon} = \begin{bmatrix}\tilde{\varepsilon}_1\\ \tilde{\varepsilon}_2\\ \vdots\\ \tilde{\varepsilon}_n\end{bmatrix}_{nt\text{ x }1}$$

response $\tilde{y}$ is a sized vector nt x 1. $\tilde{\delta}$ is a vector that contains sized parameters $\tilde{\beta}$ and $\tilde{\gamma}$ sized $(np+nrp)$ x 1, and $\tilde{\varepsilon}$ is error vector. If $\tilde{\varepsilon}\sim N(\tilde{0},\sigma^2\mathbf{W})$, so $\tilde{y}\sim N\left(\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta},\sigma^2\mathbf{W}\right)$, when

$$\tilde{0} = \begin{pmatrix}\tilde{0}\\ \tilde{0}\\ \vdots\\ \tilde{0}\end{pmatrix} \text{dan } \sigma^2\mathbf{W} = \begin{bmatrix}\sigma^2\mathbf{W_1} & \mathbf{0} & \cdots & \mathbf{0}\\ \mathbf{0} & \sigma^2\mathbf{W_2} & \cdots & \mathbf{0}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{0} & \mathbf{0} & \cdots & \sigma^2\mathbf{W}_n\end{bmatrix}$$

the simultaneous hypothesis formulation used to test the significance of non-parametric spline truncated regression model parameters on longitudinal data is:

$H_0: \tilde{\delta} = 0$

H_1: at least one component inside $\tilde{\delta} \neq 0$

where

$$\tilde{\delta} = \begin{pmatrix}\beta_{11},\ldots,\beta_{m1},\ldots,\beta_{1p},\ldots,\beta_{mp}\\ \beta_{12},\ldots,\beta_{m2},\ldots,\beta_{2p},\ldots,\beta_{mp}\\ \vdots\\ \beta_{1n},\ldots,\beta_{mn},\ldots,\beta_{np},\ldots,\beta_{mp}\\ \gamma_{11},\ldots,\gamma_{r1},\ldots,\gamma_{1p},\ldots,\gamma_{rp}\\ \vdots\\ \gamma_{1n},\ldots,\gamma_{rn},\ldots,\gamma_{np},\ldots,\gamma_{rp}\end{pmatrix}'$$

parameter space below H (Ω) as follows:

$$\Omega = \{\tilde{\delta} = (\tilde{\beta}_1,\tilde{\beta}_2,\ldots,\tilde{\beta}_n,\tilde{\gamma}_1,\tilde{\gamma}_2,\ldots,\tilde{\gamma}_n),\sigma^2\mathbf{W}\} \qquad (7)$$

whereas, the parameter space below $H_0(\omega)$ as follows:

$$\omega = \{(\tilde{\delta},\sigma^2\mathbf{W}),\tilde{\delta} = \tilde{0}\} = \{\sigma^2\mathbf{W}\} \qquad (8)$$

To get the parameter estimation below $H(\Omega)$ pada data longitudinal one way is to use WLS method. Based on the equation (6), then the model under space $H(\Omega)$ is:

$$\tilde{y} = \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_\Omega + \tilde{\varepsilon}$$

so, the equation as follows:

$$\tilde{\varepsilon} = \tilde{y} - \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_\Omega$$

Next is give the equation of the sum of squares error with the weighting matrix $\mathbf{W}$ as follows:

$$\begin{aligned}\tilde{\varepsilon}'\mathbf{W}^{-1}\tilde{\varepsilon} &= \left(\tilde{y} - \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_\Omega\right)'\mathbf{W}_\Omega^{-1}\left(\tilde{y} - \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_\Omega\right)\\ &= \left(\tilde{y}' - \tilde{\delta}_\Omega{}'\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)\mathbf{W}_\Omega^{-1}\left(\tilde{y} - \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_\Omega\right)\\ &= \tilde{y}'\mathbf{W}_\Omega^{-1}\tilde{y} - \tilde{\delta}_\Omega{}'\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\tilde{y} - \tilde{y}'\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_\Omega +\\ &\quad \tilde{\delta}_\Omega{}'\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_\Omega\\ &= \tilde{y}'\mathbf{W}_\Omega^{-1}\tilde{y} - 2\tilde{\delta}_\Omega{}'\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\tilde{y} +\\ &\quad \tilde{\delta}_\Omega{}'\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_\Omega\\ &= Q(\tilde{\delta}_\Omega)\end{aligned}$$

From equation (7) will be partially derived from $\tilde{\delta}_\Omega$ to obtain an estimate of the parameters below space Ω, as follows:

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$\frac{\partial(Q(\tilde{\delta}_\Omega))}{\partial(\tilde{\delta}_\Omega)} = \frac{\partial\left(\tilde{y}'\mathbf{W}_\Omega^{-1}\tilde{y} - 2\tilde{\delta}_\Omega{}'\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\tilde{y} + \tilde{\delta}_\Omega{}'\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_\Omega\right)}{\partial(\tilde{\delta}_\Omega)}$

$= -2\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\tilde{y} + 2\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_\Omega$

if the result of the partial derivative is equal to zero, then an equation of parameter estimator below space Ω is as follows:

$$-2\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\tilde{y} + 2\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_\Omega = 0$$

$$\hat{\tilde{\delta}}_\Omega = \left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\tilde{y}$$

If the likelihood function is given for the parameter space Ω as follows:

$$L\left(\tilde{\delta}_\Omega,\mathbf{W}_\Omega\right) = \prod_{i=1}^n f(\tilde{y}_i)$$
$$= \prod_{i=1}^n \frac{1}{2\pi|\mathbf{W}_{\Omega i}|^{1/2}} e^{-\frac{1}{2}\left(\tilde{y}_i - \mathbf{T_i}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_{\Omega i}\right)'\mathbf{W}_{\Omega i}^{-1}\left(\tilde{y}_i - \mathbf{T_i}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_{\Omega i}\right)}$$
$$= (2\pi|\mathbf{W}_\Omega|)^{-\frac{n}{2}} e^{-\frac{1}{2}\left(\tilde{y} - \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_\Omega\right)'\mathbf{W}_\Omega^{-1}\left(\tilde{y} - \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_\Omega\right)}$$
$$= (2\pi|\mathbf{W}_\Omega|)^{-\frac{n}{2}} e^{-\frac{1}{2}\left(\tilde{y}'\mathbf{W}_\Omega^{-1}\tilde{y} - 2\tilde{\delta}_\Omega{}'\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\tilde{y} + \tilde{\delta}_\Omega{}'\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_\Omega\right)} \quad (9)$$

then, if the equation (9) lowered against $\mathbf{W}_\Omega$, the estimator equation will be obtained for $\widehat{\mathbf{W}}_\Omega$ as follows:

$$\widehat{\mathbf{W}}_\Omega = \frac{\left(\tilde{y} - \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\hat{\tilde{\delta}}_\Omega\right)'\left(\tilde{y} - \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\hat{\tilde{\delta}}_\Omega\right)}{n} \quad (10)$$

If $\left(\tilde{y} - \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\hat{\tilde{\delta}}_\Omega\right) = A$ and $\left(\tilde{y} - \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\hat{\tilde{\delta}}_\Omega\right)' = A'$, then substituting the equation (10) into equation (9) then the maximum value of the likelihood function becomes:

$$\max_\Omega L\left(\hat{\tilde{\delta}}_\Omega,\widehat{\mathbf{W}}_\Omega\right) = \left(2\pi|\widehat{\mathbf{W}}_\Omega|\right)^{-\frac{n}{2}} e^{-\frac{n}{2}\left(\mathrm{A}'(\mathrm{A}'\mathrm{A})^{-1}\mathrm{A}\right)} \quad (11)$$

by following the operating properties of that matrix $\mathrm{A}'(\mathrm{A}')^{-1} = \mathrm{I}$ dan $\mathrm{A}^{-1}\mathrm{A} = \mathrm{I}$, then equation (11) become:

$$\max_\Omega L\left(\hat{\tilde{\delta}}_\Omega,\widehat{\mathbf{W}}_\Omega\right) = \left(2\pi|\widehat{\mathbf{W}}_\Omega|\right)^{-\frac{n}{2}} e^{-\frac{n}{2}}$$

The next step is to get the parameter estimate below $H_0(\omega)$ by using the Function methods Multiplier Lagrange (LM), because there are conditions (constraints). Given LM function as follows:

$$F\left(\tilde{\delta}_\omega,\theta\right) = V\left(\tilde{\delta}_\omega\right) + 2\theta'(\tilde{\delta}_\omega) \quad (12)$$

where:

$$V\left(\tilde{\delta}_\omega\right) = \left(\tilde{y} - \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_\omega\right)'\mathbf{W}_\omega^{-1}\left(\tilde{y} - \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_\omega\right) \quad (13)$$

with condition $(\tilde{\delta}_\omega)$. So, equation (13) will be described as follows:

$$V\left(\tilde{\delta}_\omega\right) = \left(\tilde{y} - \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_\omega\right)'\mathbf{W}_\omega^{-1}\left(\tilde{y} - \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_\omega\right)$$
$$= \tilde{y}'\mathbf{W}_\omega^{-1}\tilde{y} - \tilde{\delta}_\omega{}'\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\tilde{y} - \tilde{y}'\mathbf{W}_\omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_\omega + \tilde{\delta}_\omega{}'\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_\omega$$
$$= \tilde{y}'\mathbf{W}_\omega^{-1}\tilde{y} - 2\tilde{\delta}_\omega{}'\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\tilde{y} + \tilde{\delta}_\omega{}'\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_\omega \quad (14)$$

Furthermore, by substituting equation (14) into equation (12), we obtain the following equation:

$$F\left(\tilde{\delta}_\omega,\theta\right) = \tilde{y}'\mathbf{W}_\omega^{-1}\tilde{y} - 2\tilde{\delta}_\omega{}'\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\tilde{y} + \tilde{\delta}_\omega{}'\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_\omega + 2\theta'(\tilde{\delta}_\omega) \quad (15)$$

From equation (15) will be derived partially to $\tilde{\delta}_\omega$ to obtain an estimate of the parameter under space ω, which is as follows:

$$\frac{\partial\left(F(\tilde{\delta}_\omega,\theta)\right)}{\partial(\tilde{\delta}_\omega)} = -2\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\tilde{y} + 2\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_\omega + 2\theta \quad (16)$$

if equation (16) is equal to zero, then the equation becomes:

$$-2\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\tilde{y} + 2\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_\omega + 2\theta = 0$$
$$\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_\omega = \mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\tilde{y} - \theta$$

so, get $\hat{\tilde{\delta}}_\omega$ as follows:

$$\hat{\tilde{\delta}}_\omega = \left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\tilde{y} - \theta\right)$$
$$= \left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\tilde{y} - \left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\theta$$
$$= \hat{\tilde{\delta}}_\Omega - \left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\theta \quad (17)$$

then, if equation (15) is lowered to θ, then the equation will be obtained:

$$\frac{\partial\left(F(\tilde{\delta}_\omega,\theta)\right)}{\partial(\theta)} = 2\tilde{\delta}_\omega \quad (18)$$

if equation (18) is equal to zero, then it will be:

$$\hat{\tilde{\delta}}_\omega = 0 \quad (19)$$

then equation (19) is substituted into equation (17) to obtain the value θ as follows:

$$0 = \hat{\tilde{\delta}}_\Omega - \left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\theta$$
$$\left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\theta = \hat{\tilde{\delta}}_\Omega$$
$$\theta = \left(\left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\right)^{-1}\hat{\tilde{\delta}}_\Omega$$
$$\theta = \mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\hat{\tilde{\delta}}_\Omega$$

so, equation (17) becomes:

$$\hat{\tilde{\delta}}_\omega = \hat{\tilde{\delta}}_\Omega - \left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\hat{\tilde{\delta}}_\Omega$$

If the likelihood function is given below the parameter space ω as follows:

$$L\left(\tilde{\delta}_\omega,\mathbf{W}_\omega\right) = \prod_{i=1}^n f(\tilde{y}_i)$$
$$= \prod_{i=1}^n \frac{1}{2\pi|\mathbf{W}_{\omega i}|^{1/2}} e^{-\frac{1}{2}\left(\tilde{y}_i - \mathbf{T_i}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_{\omega i}\right)'\mathbf{W}_{\omega i}^{-1}\left(\tilde{y}_i - \mathbf{T_i}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_{\omega i}\right)}$$
$$= (2\pi|\mathbf{W}_\omega|)^{-\frac{n}{2}} e^{-\frac{1}{2}\left(\tilde{y} - \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_\omega\right)'\mathbf{W}_\omega^{-1}\left(\tilde{y} - \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_\omega\right)}$$
$$= (2\pi|\mathbf{W}_\omega|)^{-\frac{n}{2}} e^{-\frac{1}{2}\left(\tilde{y}'\mathbf{W}_\omega^{-1}\tilde{y} - 2\tilde{\delta}_\omega{}'\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\tilde{y} + \tilde{\delta}_\omega{}'\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_\omega\right)} \quad (20)$$

$$\log L\left(\tilde{\delta}_\omega,\mathbf{W}_\omega\right) = -\frac{n}{2}\log(2\pi|\mathbf{W}_\omega|) - \frac{1}{2}\left(\tilde{y}'\mathbf{W}_\omega^{-1}\tilde{y} - 2\tilde{\delta}_\omega{}'\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\tilde{y} + \tilde{\delta}_\omega{}'\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\tilde{\delta}_\omega\right) \quad (21)$$

then if equation (21) is partially derived $\mathbf{W}_\omega$, the estimator equation will be obtained for $\widehat{\mathbf{W}}_\omega$ as follows:

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$$\widehat{\mathbf{W}}_\omega = \frac{\left(\tilde{y}-\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\hat{\tilde{\delta}}_\omega\right)'\left(\tilde{y}-\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\hat{\tilde{\delta}}_\omega\right)}{n} \qquad (22)$$

If $\left(\tilde{y}-\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\hat{\tilde{\delta}}_\omega\right) = \mathrm{B}$ and $\left(\tilde{y}-\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\hat{\tilde{\delta}}_\omega\right)' = \mathrm{B}'$, then by substituting equation (22) into equation (20) then the maximum value of the likelihood function becomes:

$$\max_\omega L\left(\hat{\tilde{\delta}}_\omega, \widehat{\mathbf{W}}_\omega\right) = (2\pi|\widehat{\mathbf{W}}_\omega|)^{-\frac{n}{2}} e^{-\frac{n}{2}\left(\mathrm{B}'(\mathrm{B}'\mathrm{B})^{-1}\mathrm{B}\right)} \qquad (23)$$

by following the operating properties of that matrix $\mathrm{B}'(\mathrm{B}')^{-1} = \mathrm{I}$ dan $\mathrm{B}^{-1}\mathrm{B} = \mathrm{I}$, then equation (23) becomes:

$$\max_\omega L\left(\hat{\tilde{\delta}}_\omega, \widehat{\mathbf{W}}_\omega\right) = (2\pi|\widehat{\mathbf{W}}_\omega|)^{-\frac{n}{2}} e^{-\frac{n}{2}}$$

4.3 Likelihood Ratio for Test Statistic

$$\lambda(z, z(\tilde{k}), y) = \frac{L(\hat{\omega})}{L(\hat{\Omega})} = \frac{(2\pi|\widehat{\mathbf{W}}_\omega|)^{-\frac{n}{2}} e^{-\frac{n}{2}}}{(2\pi|\widehat{\mathbf{W}}_\Omega|)^{-\frac{n}{2}} e^{-\frac{n}{2}}} = \left(\frac{|\widehat{\mathbf{W}}_\omega|}{|\widehat{\mathbf{W}}_\Omega|}\right)^{-\frac{n}{2}}$$

$$= \left(\frac{\left(\tilde{y}-\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\hat{\tilde{\delta}}_\omega\right)'\left(\tilde{y}-\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\hat{\tilde{\delta}}_\omega\right)}{\left(\tilde{y}-\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\hat{\tilde{\delta}}_\Omega\right)'\left(\tilde{y}-\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\hat{\tilde{\delta}}_\Omega\right)}\right)^{-\frac{n}{2}} \qquad (24)$$

Then do the translation of the numerator by considering G to be:

$$\mathrm{G} = \left(\tilde{y}-\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\hat{\tilde{\delta}}_\omega\right)'\left(\tilde{y}-\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\hat{\tilde{\delta}}_\omega\right)$$

then substituting $\left(-\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\hat{\tilde{\delta}}_\Omega + \mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\hat{\tilde{\delta}}_\Omega\right)$ into G, then it will be:

$$\mathrm{G} = \left(\tilde{y}-\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\hat{\tilde{\delta}}_\Omega\right)'\left(\tilde{y}-\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\hat{\tilde{\delta}}_\Omega\right) + \mathrm{P}\left(\hat{\tilde{\delta}}_\Omega - \hat{\tilde{\delta}}_\omega\right) + \left(\hat{\tilde{\delta}}_\Omega - \hat{\tilde{\delta}}_\omega\right)'\mathbf{Q} + \left(\hat{\tilde{\delta}}_\Omega - \hat{\tilde{\delta}}_\omega\right)'\mathbf{T}'(\mathbf{Z},\mathbf{Z}(\tilde{k}))\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\left(\hat{\tilde{\delta}}_\Omega - \hat{\tilde{\delta}}_\omega\right) \qquad (25)$$

the component of the second and third segments in equation (25) will then be described $\mathrm{P} = \left(\tilde{y}-\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\hat{\tilde{\delta}}_\Omega\right)'\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))$ and $\mathrm{Q} = \mathbf{T}'(\mathbf{Z},\mathbf{Z}(\tilde{k}))\left(\tilde{y}-\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\hat{\tilde{\delta}}_\Omega\right)$ by distributing $\hat{\tilde{\delta}}_\Omega$ as follows:

$$\mathrm{P} = \left(\tilde{y} - \mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\left(\mathbf{T}'(\mathbf{Z},\mathbf{Z}(\tilde{k}))\mathbf{W}_\omega^{-1}\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\right)^{-1}\left(\mathbf{T}'(\mathbf{Z},\mathbf{Z}(\tilde{k}))\mathbf{W}_\omega^{-1}\tilde{y}\right)\right)'\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))$$

$$= \tilde{y}'\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k})) - \tilde{y}'\mathbf{W}_\omega^{-1}\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\left(\mathbf{T}'(\mathbf{Z},\mathbf{Z}(\tilde{k}))\mathbf{W}_\omega^{-1}\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\right)^{-1}\mathbf{T}'(\mathbf{Z},\mathbf{Z}(\tilde{k}))\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))$$

$$= \tilde{y}'\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k})) - \tilde{y}'\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k})) = 0$$

and

$$\mathrm{Q} = \mathbf{T}'(\mathbf{Z},\mathbf{Z}(\tilde{k}))\left(\tilde{y} - \mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\left(\mathbf{T}'(\mathbf{Z},\mathbf{Z}(\tilde{k}))\mathbf{W}_\omega^{-1}\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\right)^{-1}\left(\mathbf{T}'(\mathbf{Z},\mathbf{Z}(\tilde{k}))\mathbf{W}_\omega^{-1}\tilde{y}\right)\right)$$

$$= \mathbf{T}'(\mathbf{Z},\mathbf{Z}(\tilde{k}))\tilde{y} - \mathbf{T}'(\mathbf{Z},\mathbf{Z}(\tilde{k}))\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\left(\mathbf{T}'(\mathbf{Z},\mathbf{Z}(\tilde{k}))\mathbf{W}_\omega^{-1}\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\right)^{-1}\left(\mathbf{T}'(\mathbf{Z},\mathbf{Z}(\tilde{k}))\mathbf{W}_\omega^{-1}\tilde{y}\right)$$

$$= \mathbf{T}'(\mathbf{Z},\mathbf{Z}(\tilde{k}))\tilde{y} - \mathbf{T}'(\mathbf{Z},\mathbf{Z}(\tilde{k}))\tilde{y} = 0$$

then the equation G will be substituted to the numerator segment in equation (24) as follows:

$$\lambda(z, z(\tilde{k}), y) = \left(\frac{\left(\tilde{y}-\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\hat{\tilde{\delta}}_\Omega\right)'\left(\tilde{y}-\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\hat{\tilde{\delta}}_\Omega\right) + \hat{\tilde{\delta}}_\Omega{}'\left(\left(\mathbf{T}'(\mathbf{Z},\mathbf{Z}(\tilde{k}))\mathbf{W}_\omega^{-1}\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\right)^{-1}\right)^{-1}\hat{\tilde{\delta}}_\Omega}{\left(\tilde{y}-\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\hat{\tilde{\delta}}_\Omega\right)'\left(\tilde{y}-\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\hat{\tilde{\delta}}_\Omega\right)}\right)^{-\frac{n}{2}}$$

$$= \left(1 + \frac{\hat{\tilde{\delta}}_\Omega{}'\left(\left(\mathbf{T}'(\mathbf{Z},\mathbf{Z}(\tilde{k}))\mathbf{W}_\omega^{-1}\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\right)^{-1}\right)^{-1}\hat{\tilde{\delta}}_\Omega}{\left(\tilde{y}-\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\hat{\tilde{\delta}}_\Omega\right)'\left(\tilde{y}-\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\hat{\tilde{\delta}}_\Omega\right)}\right)^{-\frac{n}{2}}$$

$$= \left(\frac{1}{1 + \frac{\hat{\tilde{\delta}}_\Omega{}'\left(\left(\mathbf{T}'(\mathbf{Z},\mathbf{Z}(\tilde{k}))\mathbf{W}_\omega^{-1}\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\right)^{-1}\right)^{-1}\hat{\tilde{\delta}}_\Omega}{\left(\tilde{y}-\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\hat{\tilde{\delta}}_\Omega\right)'\left(\tilde{y}-\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\hat{\tilde{\delta}}_\Omega\right)}}\right)^{-\frac{n}{2}}$$

$$= \left(\frac{1}{1+\frac{\mathrm{M}_1}{\mathrm{M}_2}}\right)^{-\frac{n}{2}} \qquad (26)$$

Based on the equation (26), we can find the test statistic of the hypothesis H_0: $\tilde{\delta} = 0$ against H_1: there is at least one parameter component inside $\tilde{\delta} \neq 0$ with process as follows:

$$\left(\frac{1}{1+\frac{\mathrm{M}_1}{\mathrm{M}_2}}\right)^{-\frac{n}{2}} < k$$

$$\mathrm{V} > k^*$$

so, it can be stated that k^* is a test statistic, where

$$k^* = \left(\left(\frac{1}{\mathrm{k}}\right)^{\frac{n}{2}} - 1\right)\frac{(ntp - (np + nrp))}{(np + nrp)}$$

4.4 Distribution of Testing Statistics

Based on equation (26) we obtain an equation denoted by Q as below:

$$\mathrm{Q} = \frac{\mathrm{M}_1}{\mathrm{M}_2} = \frac{\hat{\tilde{\delta}}_\Omega{}'\left(\left(\mathbf{T}'(\mathbf{Z},\mathbf{Z}(\tilde{k}))\mathbf{W}_\omega^{-1}\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\right)^{-1}\right)^{-1}\hat{\tilde{\delta}}_\Omega}{\left(\tilde{y}-\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\hat{\tilde{\delta}}_\Omega\right)'\left(\tilde{y}-\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\hat{\tilde{\delta}}_\Omega\right)} \qquad (27)$$

The next step is to get the distribution of the test statistic, then what needs to be done is to describe the equation M_1 and M_2 in equation (27). The following is a translation process for the equations M_1, is:

$$\mathrm{M}_1 = \hat{\tilde{\delta}}_\Omega{}'\left(\left(\mathbf{T}'(\mathbf{Z},\mathbf{Z}(\tilde{k}))\mathbf{W}_\omega^{-1}\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\right)^{-1}\right)^{-1}\hat{\tilde{\delta}}_\Omega$$

$$= \tilde{y}'\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\left(\mathbf{T}'(\mathbf{Z},\mathbf{Z}(\tilde{k}))\mathbf{W}_\Omega^{-1}\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\right)^{-1}\mathbf{T}'(\mathbf{Z},\mathbf{Z}(\tilde{k}))\mathbf{W}_\Omega^{-1}\tilde{y} \qquad (28)$$

From equation (28), if:

$$\mathbf{A} = \mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\left(\mathbf{T}'(\mathbf{Z},\mathbf{Z}(\tilde{k}))\mathbf{W}_\Omega^{-1}\mathbf{T}(\mathbf{Z},\mathbf{Z}(\tilde{k}))\right)^{-1}\mathbf{T}'(\mathbf{Z},\mathbf{Z}(\tilde{k}))\mathbf{W}_\Omega^{-1}$$

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And it will be proven that matrix **A** is symmetrical and idempotent. Matrix **A** is symmetrical if $\mathbf{A}' = \mathbf{A}$. The proof is:

$$\mathbf{A}' = \left(\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\right)'$$

$$= \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}$$

$$= \mathbf{A}\ \textbf{(proven)} \qquad (29)$$

and,

$$\mathbf{A}^2 = \left(\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\right)^2$$

$$= \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\ x$$
$$\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}$$

$$= \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}$$

$$= \mathbf{A}\ \textbf{(proven)} \qquad (30)$$

From equations (29) and (30), it has been proven that the matrix **A** is symmetric and idempotent then M_1 also symmetrical and idempotent, so it can be stated that:

$$\frac{\mathrm{M}_1}{\boldsymbol{\sigma}^2}\sim\chi^2_{(r_1,\tilde{\mu}'A\tilde{\mu}/2\boldsymbol{\sigma}^2)}$$

then want to get the value r_1 dan $\tilde{\mu}'A\tilde{\mu}/2\boldsymbol{\sigma}^2$. Because matrix **A** is symmetric and idempotent then $r_1 = rank(\mathbf{A}) = tr(\mathbf{A})$.

$$tr(\mathbf{A}) = tr\left(\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\right)$$

If $\mathbf{B} = \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}$ and $\mathbf{C} = \mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}$, t $tr(\mathbf{A}) = tr(\mathbf{BC}) = tr(\mathbf{CB})$, so,

$$tr(\mathbf{A}) = tr\left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\right)$$
$$= tr(\mathbf{I}_{(np+nrp)})$$
$$= (np+nrp)$$

So, $r_1 = rank(\mathbf{A}) = tr(\mathbf{A}) = (np+nrp)$.

Next will looking for value from $\tilde{\mu}'A\tilde{\mu}/2\boldsymbol{\sigma}^2$ as follows:

$$\tilde{\mu}'A\tilde{\mu} = \left(\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\hat{\tilde{\delta}}_\omega\right)'\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\hat{\tilde{\delta}}_\omega$$
$$= \hat{\tilde{\delta}}_\omega{}'\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\hat{\tilde{\delta}}_\omega$$
$$= \hat{\tilde{\delta}}_\Omega{}'\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\hat{\tilde{\delta}}_\Omega - \hat{\tilde{\delta}}_\Omega{}'\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\hat{\tilde{\delta}}_\Omega$$
$$= 0$$

so, it can be stated that:

$$\frac{\mathrm{M}_1}{\boldsymbol{\sigma}^2}\sim\chi^2_{(np+nrp)}$$

The next stage is to describe the segment M_2 as follows:

$$\mathrm{M}_2 = \left(\tilde{y} - \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\hat{\tilde{\delta}}_\Omega\right)'\left(\tilde{y} - \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\hat{\tilde{\delta}}_\Omega\right)$$
$$= \left(\tilde{y}' - \hat{\tilde{\delta}}_\Omega{}'\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)\left(\tilde{y} - \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\hat{\tilde{\delta}}_\Omega\right)$$
$$= \tilde{y}'\tilde{y} - 2\hat{\tilde{\delta}}_\Omega{}'\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right) + \hat{\tilde{\delta}}_\Omega{}'\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\hat{\tilde{\delta}}_\Omega$$
$$= \tilde{y}'\left[\mathbf{I} - \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right]\tilde{y}$$
$$= \tilde{y}'\mathbf{B}\tilde{y} \qquad (31)$$

From equation (31) can be specified if:

$$\mathbf{B} = \mathbf{I} - \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)$$

then it will be proven that **B** symmetrical and idempotent matrix. Matrix **B** symmetric if $\mathbf{B}' = \mathbf{B}$, and here is the translation.

$$\mathbf{B}' = \left(\mathbf{I} - \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)'$$
$$= \mathbf{I} - \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)$$
$$= \mathbf{B}\ \textbf{(proven)}$$

dan

$$\mathbf{B}^2 = \left(\mathbf{I} - \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^2$$
$$= \mathbf{I} - \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)$$
$$= \mathbf{B}\ \textbf{(proven)}$$

so, it proved that **B** is idempotent matrix. Because M_2 is symmetrical and idempotent, then it can be stated that:

$$\frac{\mathrm{M}_2}{\boldsymbol{\sigma}^2}\sim\chi^2_{(r_2,\tilde{\mu}'A\tilde{\mu}/2\boldsymbol{\sigma}^2)}$$

Next will find the value for r_2 and $\tilde{\mu}'A\tilde{\mu}/2\boldsymbol{\sigma}^2$. Because **B** adalah symmetrical and idempotent matrix, then $r_2 = rank(\mathbf{B}) = tr(\mathbf{B})$.

$$tr(\mathbf{B}) = tr\begin{pmatrix}\mathbf{I} - \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1} \\ \mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\end{pmatrix}$$
$$= tr(\mathbf{I}) - tr\begin{pmatrix}\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right) \\ \left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\end{pmatrix}$$
$$= tr(\mathrm{I}_{\boldsymbol{ntp}}) - tr(\mathbf{I}_{(np+nrp)})$$
$$= ntp - (np+nrp)$$

so, it is obtained that:

$r_2 = rank(\mathbf{B}) = tr(\mathbf{B}) = ntp - (np+nrp)$

and then the value will be searched from $\tilde{\mu}'\mathbf{B}\tilde{\mu}/2\boldsymbol{\sigma}^2$ as follows:

$$\frac{\tilde{\mu}'\mathbf{B}\tilde{\mu}}{2\boldsymbol{\sigma}^2} = \frac{\hat{\tilde{\delta}}'\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\left(\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\hat{\tilde{\delta}}\right) - \hat{\tilde{\delta}}'\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\left(\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\hat{\tilde{\delta}}\right)}{2\boldsymbol{\sigma}^2}$$
$$= 0$$

so, it can be stated that:

$$\frac{\mathrm{M}_2}{\boldsymbol{\sigma}^2}\sim\chi^2_{(ntp-(np+nrp))}$$

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$$\mathbf{AB} = \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{x}$$

$$\mathbf{I}-\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)$$

$$= \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1} +$$

$$-\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}$$

$$\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)$$

$$= \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1} +$$

$$-\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}$$

$$= 0$$

because $\mathbf{AB}=\mathbf{0}$, then M_1 and M_2 independent. From some elaboration of the above equation, the following results are obtained:

1. $\frac{\mathrm{M}_1}{\sigma^2} \sim \chi^2_{(np+nrp)}$
2. $\frac{\mathrm{M}_2}{\sigma^2} \sim \chi^2_{(ntp-(np+nrp))}$
3. M_1 and M_2 independently

so, based on Rencher (2007) which states that if u is $\chi^2_{(p)}$, v is $\chi^2_{(q)}$, also u and v independent, then:

$$\mathrm{V} = \frac{u/p}{v/q} \sim F_{(p,q)}$$

$$\mathrm{V} = \frac{\mathrm{M}_1/(np+nrp)}{\mathrm{M}_2/(ntp-(np+nrp))} \sim F_{((np+nrp),ntp-(np+nrp))}$$

with

$$\mathrm{M}_1 = \tilde{y}'\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\tilde{y}$$

$$\mathrm{M}_2 = \tilde{y}'\left[\mathbf{I} - \mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right]\tilde{y}$$

4.5 Rejection Area of Hypothesis Null

Rejection area $\lambda < k$, where $\lambda < k < 1$, and k is a constant, so:

$$\lambda(z, z(\tilde{k}), y) = \frac{L(\widehat{\omega})}{L(\widehat{\Omega})} < k$$

Based on statistic test, then the critical area for the test $H_0: \tilde{\delta} = 0$, dan H_1: there is at least one parameter component inside $\tilde{\delta} \neq 0$ is

$$\mathrm{V} = \left\{(z_{ij1}, z_{ij2}, \dots, z_{ijp});\ \mathrm{V} > k^*\right\}$$

If given the level of trust α, then:

$$\alpha = P(Reject\ H_0 | H_0\ True) = P(\mathrm{V} > k^* | \tilde{\delta}_i = 0)$$

with

$$\mathrm{V} = \frac{\mathrm{M}_1/(np+nrp)}{\mathrm{M}_2/(ntp-(np+nrp))} \sim F_{((np+nrp),ntp-(np+nrp))}$$

so, critical areas to resist H_0 provided if the test statistic V ($F\ calculated$) greater than $k^*(F\ table)$.

5 Conclusion

Based on the description of analysis and discussion in this study, it can be drawn some conclusions as follows:

a. The estimation of spline truncated nonparametric regression parameters on the parameter space below Ω is

$$\hat{\tilde{\delta}}_\Omega = \left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1}\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\Omega^{-1}\tilde{y}$$

and the parameter estimate under space ω is

$$\hat{\tilde{\delta}}_\omega = \hat{\tilde{\delta}}_\Omega - \left(\mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\right)^{-1} \mathbf{T}'\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\mathbf{W}_\omega^{-1}\mathbf{T}\left(\mathbf{Z},\mathbf{Z}(\tilde{k})\right)\hat{\tilde{\delta}}_\Omega$$

b. Test statistics obtained:
$\mathrm{V} > k^*$
so, it can be stated that k^* is a test statistic, where

$$k^* = \left(\left(\frac{1}{\mathrm{k}}\right)^{\frac{n}{2}} - 1\right)\frac{(n-(np+nrp))}{(np+nrp)}$$

c. The distribution of the test statistic obtained follows the distribution $F_{((np+nrp),ntp-(np+nrp))}$

d. The rejection area H_0 is $\mathrm{V} = \left\{(z_{ij1}, z_{ij2}, \dots, z_{ijp});\ \mathrm{V} > k^*\right\}$. If given a confidence level $\alpha = P(\mathrm{V} > k^* | \tilde{\delta}_i = 0)$ or H_0 rejected if $\mathrm{V}(\mathrm{F_{calculated}}) > k^*(F\ tabel)$.

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